reduce the light transmission approximately 10%. Ordinarily, less solid than this will be present in the sample at the last temperature step.

To determine the eutectic points or phase transitions, heating curves in the proper temperature region using very small temperature gradients (0.1° per minute or less) are employed. Temperature flats are obtained when the rate of heat input had been reduced to a point such that the eutectic melting or transition can absorb the heat completely.

Faraday-Cup Monitors for High-Energy Electron Beams*

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(Received June 11, 1956)

Two Faraday-cup electron collectors have been developed which are capable of measuring the absolute integrated beam current of the Stanford linear accelerators to better than 0.5% at electron energies ranging from 4 to 300 Mev. A description of these instruments is given and complete design criteria are offered which allow the extension of the range to GeV energies.

INTRODUCTION

With the successful operation of electron linear accelerators having large peak currents with small cross-sectional area, it has become evident to investigators using these machines that Faraday cups are potentially the best available absolute beam monitors. Although the initial testing of the cup may be tedious, the calibration at a given electron energy never changes except possibly from leakage currents which are readily checked and become obvious during the use of the instrument. For a given energy, the calibration of the cup as a function of peak current is linear. Such a monitor is a "primary standard" since it measures directly the quantity of interest, the integrated beam current, by transferring the collected charge to a calibrated condenser.

DESIGN PROCEDURE

A basic parameter in the design of a Faraday-cup integrator is the fraction of the incident charge which is permitted to escape, or in other words, the absolute accuracy demanded of the integrator. There are several mechanisms by which the integrated beam current may be falsified: by penetration of the electron shower, by backscatter from the mouth of the integrator, by leakage currents to ground, and by collection of ions or electrons which have been produced in the vicinity of the cup.

The Faraday cup must be made large enough so that the net charge escaping due to shower penetration is well within the design specifications. It is, however, important not to overdesign on this matter since the weight of a cup for 1-GeV electrons is of the order of a ton, depending upon the incident beam diameter and the absolute accuracy demanded of the integrator.

Three procedures for avoiding loss from backscatter have been adopted in this laboratory: First, the geometry of the front of the integrator is made reentrant as is shown in Figs. 1 and 2. This reduces the solid angle of the backscatter. Second, a permanent magnet is inserted in the mouth of the integrator so that low-energy backscattered particles are prevented from escaping. Last, the bottom of the Faraday cup where

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* The research reported here was sponsored by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
Laboratory Technical Report No.

An invaluable guide. A summary of Kantz's results for percentage energy loss by shower penetration as a function of absorber depth yields a straight line. Graphs of the logarithm of the percentage energy loss has been reached a plot of the logarithm of the percentage energy loss as a function of absorber depth.

Kantz and Hofstadter 1-3 on electron-induced showers is optimized as shown by the intersecting solid curve in Figs. 4 to 6. The curves in Figs. 5 and 6 show the depth and radius of a cylinder required to capture a given percentage of the incident electron energy.

Fig. 4. Data obtained by Kantz 1-3 showing the longitudinal development of an electron-induced shower in carbon and lead. The slope of the transition curve after the shower maximum corresponds to the minimum-absorption coefficient for gamma rays in the medium.

There will be other additional factors—such as collection of secondary electrons produced at an entrance foil—which may influence the design of an integrator depending upon the particular experimental conditions, but the problems mentioned are fundamental to all situations. The last two are of a trivial nature and are easily checked experimentally. However, shower penetration and backscatter require careful consideration and are not as readily checked experimentally.

There are several ways in which the charge loss due to shower penetration may be checked, and these will be discussed later; but for design purposes the work of Kantz and Hofstadter 1-3 on electron-induced showers is an invaluable guide. A summary of Kantz's results for carbon and lead is shown in Figs. 4 to 6. The curves in Figs. 5 and 6 show the depth and radius of a cylinder required to capture a given percentage of the incident electron energy. If the radius and depth of the Faraday cup are optimized as shown by the intersecting solid curve in Fig. 6, it is observed that after the shower maximum has been reached a plot of the logarithm of the percentage energy loss by shower penetration as a function of absorber depth yields a straight line. Graphs of the logarithm of the percentage energy loss as a function of depth in radiation lengths as taken from the Kantz isoenergetic curves are shown in Fig. 7 for various absorbing materials. From the graphs it is clear that the results may be extrapolated to much lower values of $f_e$ than it was possible for Kantz to measure. The percentage energy loss $f_e$ is given by

$$f_e = Ae^{-rx},$$

provided that

$$r = mx + b,$$

where $r$ and $x$ are the radius and depth of the absorber in radiation lengths; $A$ is a constant which depends only upon the energy of the incident electron; $\sigma$, $m$, and $b$ are constants which are dependent only upon the absorbing material. In fact, $\sigma$ is found to be equal to the minimum photon-absorption coefficient for the material; for example, for lead, $1/\sigma = 4.25$ radiation-lengths and $m$ and $b$ are found from a straight-line approximation of the solid curve in Fig. 6 to be $m = 0.85$ and $b = -2$.

Kantz's data were taken at 185 Mev. To extend his results to other energies requires that the energy dependence of the constant $A$ be known. However, this

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Fig. 6. Isoenergetic curves of Kantzl-3 for lead. These data show the dimensions of a cylinder of lead required to absorb a given fraction of the incident electron energy. A 45°-line was drawn to each isoenergetic curve from the point of intersection of its “asymptotes.” The solid curve passing through the intersections of the 45°-lines with their respective isoenergetic curves defines the “optimum” dimensions of an absorbing cylinder as used in the design of the Faraday cups.

may be obtained to a good approximation from the following theoretical consideration: A high-energy electron or gamma ray travels on the average a distance of \( \log 2 \) radiation lengths before its energy is reduced to one half of its initial value. Therefore, if the incident energy is doubled, the dimensions of the absorber need be increased only by \( \log 2 \) radiation lengths in order for the percentage energy loss \( f_e \) by penetration to remain substantially unchanged. This is equivalent to adding a term in the exponent of Eq. (1) as follows:

\[
f_e = A_{185} \exp \left[ -\sigma \left( x - \log \frac{E_0}{185} \right) \right]. \tag{2}
\]

If the incident electron energy is less than the critical energy of the absorbing material, a different approach to the problem can be taken. In this case, the energy loss by ionization is greater than the loss by radiation. This suggests that at low energies the Faraday cup should be constructed using a low-Z material such as carbon as the initial absorber followed by a high-Z material such as lead to absorb the photons and electrons which penetrate the carbon. The use of a low-Z material as the initial absorber has the additional advantage of reducing significantly the backscatter from the mouth of the cup. With such an arrangement the percentage energy loss \( f_e \) is then given to a good approximation by

\[
f_e = (1 - k)A_{185} \exp \left[ -\sigma \left( x - \log \frac{E_0}{185} \right) \right], \tag{3}
\]

where \( k \) is the fraction of the initial electron energy which is absorbed in the low-Z material.

An approximate equation for \( k \) may be derived assuming that the absorbed energy is entirely by collision losses

\[
(-dE/dl)_{\text{col}} = B \log E + C, \tag{4}
\]

where \( t \) is in radiation lengths and \( B \) and \( C \) are constants. If the primary electron is completely stopped in the initial absorber, the average collision loss is closely approximated by

\[
\langle (-dE/dl)_{\text{col}} \rangle \approx B \log E_0 - 1 + C = D. \tag{5}
\]

Then \( k \) for any thickness \( t \) is given by

\[
k \approx (D/E_0)t, \tag{6}
\]

if \( t \) does not exceed the range of the primary electron in the low-Z absorber. For the mean range of electrons having an energy in the vicinity of the critical energy \( E_e \), Wilson has derived the following formula:

\[
\text{range} \approx \log 2 \log \left( \frac{E_0}{E_0 \log 2 + 1} \right). \tag{7}
\]

Therefore, if the Faraday cup consists of carbon of a thickness \( t \) radiation lengths followed by \( x \)-radiation lengths of lead, the percentage energy loss is given by

\[
f_e = \left( 1 - \frac{D}{E_0} \right) A_{185} \exp \left[ -\sigma \left( x - \log \frac{E_0}{185} \right) \right], \tag{8}
\]

provided that the radius of the cup is determined from the expression

\[
r = 0.85x - 2 + \frac{\text{i.d.}}{2} \text{ radiation lengths}, \tag{9}
\]

Fig. 7. Curves showing the percent energy loss as a function of cylinder depth for cylinders which have the “optimum” dimensions as described in Fig. 6. These data are for electrons of 185 Mev.

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4 R. R. Wilson, Phys. Rev. 84, 100 (1951).
where i.d. is the inside diameter of the mouth of the cup.

For the purpose of current integration, the fraction of charged particles escaping is the important quantity. However, this may be related to the fraction of incident energy which is lost by the following argument: If an electron shower has reached the point where the average energy of the particles in the shower is below the critical energy for the medium, then the mean range of the remaining charged particles is less than one radiation length. If this condition prevails, particles which escape by penetration must be principally those which are produced near the surface by the penetrating gamma rays from the shower. For high-Z materials, these are gamma rays in the energy range 1 MeV ≤ k ≤ 15 MeV and so the charged particle production near the surface arises from these penetrating gamma rays primarily by pair production and Compton effect. In both of these cases the average energy of the charged particles produced is of the order k/2 where k is the photon energy.

At this degenerate stage of a shower there therefore exists an equilibrium between the number of charged particles and the number of gamma rays moving through the absorber. However, the number of charged particles escaping the surface is substantially less than the number of escaping gamma rays because of the greater penetrability of the photons. If Nγ is the number of photons escaping, then the number of escaping electrons Ne is approximately

\[ N_e / N_\gamma \leq R(\gamma)/R(\epsilon), \quad (10) \]

where \( R(\epsilon) \) is the mean range of electrons of energy \( k/2 \) and \( R(\gamma) \) is the attenuation length \( 1/\sigma \) of gamma rays of energy \( k \). One electron is produced in each Compton collision and two due to pair production, but since it is the net charge loss that is of importance, the number of effective electrons produced is less than one per gamma ray; hence the inequality sign.

The energy lost by escaping gamma rays is much greater than that lost by escaping charged particles, and therefore \( N_\gamma \) can be approximated by

\[ N_\gamma \approx \frac{J_p E_0}{100 k}, \quad (11) \]

where \( E_0 \) is the energy of the incident electron and \( k \) is the energy of the escaping gamma ray. It follows from Eqs. (10) and (11) that the percentage charged-particle loss \( f_p \) is given by

\[ f_p \approx \left( \frac{J_p E_0}{k} \right) \frac{R(\epsilon)}{R(\gamma)}. \quad (12) \]

A reasonable estimate for \( R(\epsilon) \) in lead is \( E/10 \) radiation lengths where \( E \approx k/2 \) is the average energy of the escaping electron in MeV, and \( R(\gamma) = 1/\sigma = 4.25 \) radiation lengths for \( k = 3 \) MeV in lead. Substituting in Eq. (12):

\[ f_p(Pb) = f_p(E_0/85), \quad (13) \]

where \( E_0 \) is in MeV.

Combining the results of Eqs. (8) and (13) and using the value of \( A_{185} = 175\% \) as given in Fig. 7, the percentage charged-particle loss by penetration from a Faraday cup consisting of \( f \) radiation lengths of a low-Z material backed by \( x \)-radiation lengths of lead is then given by

\[ f_p \approx 2E_0 \left( 1 - \frac{D}{E_0} \right) \exp \left[ -\sigma \left( x - \frac{E_0}{185} \right) \right] \% \quad (14) \]

The value of \( D \) for carbon is \( 10 \log E_0 + 53 \) MeV/radiation length, and \( 1/\sigma \) for lead is found from Fig. 7 to be 4.25 radiation-lengths.

The foregoing formulation is a procedure by which the minimum dimensions for a Faraday cup may be deduced. It is recommended that the particle loss should also be checked by experimental methods such as those to be described later.

Two Faraday-cup electron collectors have been constructed in this laboratory in accordance with the principles elucidated in the previous discussion. One of these was designed for electrons of energy up to 40 MeV and the other for electrons of energy up to 300 MeV.

**CONSTRUCTION**

A drawing of the 40-MeV cup is shown to scale in Fig. 1. The bottom of the cup consists of 4.5 in. of carbon followed by 2.5 in. of lead. The outside diameter is 12 in. and the inside diameter of the reentrant portion is 4 in. The over-all length is 16 in. The cup was constructed from a piece of pile graphite which was bored with a 9-in. deep hole and backed by lead, as illustrated. The electrons are attenuated primarily by ionization in the graphite and any photons produced are effectively absorbed by the lead. It is apparent now that 3.5 in. of carbon would have been as effective as 4.5 in. at 40 MeV and that the inch difference would have been much more useful in the form of lead.

The construction details of the 300-MeV cup are shown in Fig. 2. The cup was formed by pouring lead into a steel jacket which was then welded closed. The outgassing of the lead is thus not a problem in the attainment of a high vacuum. The cup rests upon a brass cradle which is insulated from the vacuum chamber by two polystyrene slabs. An aluminum foil guard electrode is placed between the two insulators and is connected to the electronics plate. This is required because the leakage currents to ground are serious when the cup is biased to several kilovolts. Insulation of the electronics plate is obtained by a polystyrene ring and insulating washers on the studs which compress the O-rings. The physical dimensions

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Footnote:

of the cup are: length, 24 in.; diameter, 14 in.; depth of hole, 17 in.; inside diameter, 5 in.

**ELECTRONICS**

The basic electronics circuit is shown in Fig. 3. It is designed to eliminate the effect of the stray capacitance of the cup to ground which is in parallel with the integrating condenser $C$. At the beginning of a run the cup is grounded and the electrometer is balanced to a null. During the integration, charge is accumulated on all the capacitances in the circuit. At the end of the run, charge of the opposite sign is applied to the lower terminal of the integrating condenser $B$ until the point $A$ is again at ground potential as determined by a null in the electrometer. All of the charge accumulated is thus transferred to the integrating condenser $C$ and the potential developed across it is precisely the potential applied to the terminal $B$ which can be measured with a precision potentiometer. In practice, one can use a slideback voltage which is applied continuously so that the cup is never far from ground potential and possible leakage problems are minimized.

The condensers used are of a low-leakage type* and are calibrated to an accuracy of better than 0.3% by depositing a known charge on them by means of a constant current source.

* Polystyrene dielectric condensers manufactured by the J. E. Fast Company, Chicago, Illinois.

The circuit diagram for the electrometer used in conjunction with the small integrator is shown in Fig. 8. All of the electronics are exterior to the collector which allows flexibility in the choice of integrating condensers.

Several modifications to the basic circuit were required for use with the 300-Mev cup because of long distances from the target stations to the counting rooms. Because of the large field gradients in the neighborhood of the linear accelerator due to inadequate shielding of the pulse cages, as much of the electronics as is practicable is mounted within the integrator vacuum chamber. While this results in some loss of flexibility, it eliminates the danger of pickup on high-impedance leads from the experimental station to the counting rooms.

Figure 9 shows the circuit diagram for the large-cup electronics except for the bias supply. The components surrounded by the dashed line are within the integrator vacuum. To assure circuit stability, complete isolation of the slideback circuit and the plate lead from the filament lead was found necessary. This was accomplished by separate vacuum feed-through connectors and separate cables.

The entire electronics plate at the integrator and the control chassis in the counting room must be completely insulated from ground when the cup is biased. The bias voltage supply must be extremely stable and monitored continuously since a small change in the bias will be...
reflected by a large spurious response in the integrating circuit.

TEST PROCEDURE AND PERFORMANCE

The testing of an absolute measuring instrument depends upon establishing the independence of the result of the measurement on the variables of the measuring instrument. Once the physical dimensions of the lead cup have been fixed, the remaining variables are (1) the sign and magnitude of the bias voltage to be applied between the cup and the grounded vacuum shell, and (2) the pressure within the shell. Determination of the effect of varying these parameters on the measurement requires that a second monitor be available. The basic requirement of this monitor is stability. Since one is attempting to determine a difference between a number of measurements, an absolute standard is not required but the monitor must be reproducible over the period of the time required for the measurements. All tests made on the 40-Mev integrator and those tests made at 35 Mev on the 300-Mev integrator were conducted with a hydrogen-filled ion chamber. The remainder of the tests on the 300-Mev integrator were conducted with a secondary-emission monitor.

40-Mev Integrator

Using an ion chamber operated under fixed conditions as a beam monitor, the radiation escaping from the back and sides of the cup was measured with a thimble-type Roentgen meter. At 35 Mev, the escaping radiation was easily measurable, but even under the most conservative assumptions that the ionization in the thimble was caused by minimum-ionization particles of a single sign and all traveling in one direction, the number of escaping secondaries was estimated to be at most 0.25% of the incident beam. Substituting in Eq. (14), one arrives at 0.5% which is in very good agreement.

The low-energy secondary particles entering or escaping the mouth or walls of the Faraday cup were checked by comparing the monitor-to-cup response \( R \) as a function of positive and negative bias voltage. The data are shown in Fig. 10. The crosses show the results obtained with the monitor and Faraday cup positioned as shown in Fig. 1. Curves of this type were taken at primary energies of from 2 to 35 Mev, and the shape of the curves was found to be essentially independent of energy. Some of the negative bias effect was due to secondaries being scattered into the cup from the entrance Mylar foil. By increasing the distance between the foil and the cup entrance, the foil effect was found to account for about half of the negative bias effect. The experimental points labeled with circles show the reduction in bias effect when the entrance foil is moved away from the cup. Some of the remaining effect arises from ionization occurring in the vicinity of the integrating electrometer. Because of the bias voltage, this charge is collected by the electrometer. It is of equal magnitude but of opposite sign depending upon the

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![Fig. 9. Circuit diagram of the electrometer used with the 300-Mev cup. Voltage slideback is accomplished manually.](image)

![Fig. 10. Bias curve for the 40-Mev cup.](image)

![Fig. 11. Bias curves for the 300-Mev cup taken at 35 Mev showing the reduction of backscatter when the initial absorber is carbon.](image)
sign of the applied bias voltage, and was measured by disconnecting the Faraday cup from the circuit.

As a result of these measurements, it is believed that the error due to secondaries of energy less than 900 v is certainly less than 0.6%. During actual use the cup is operated at zero bias. Errors due to higher energy secondaries which are knocked from the foils and from high-energy backscattered particles from the mouth of the cup are not measured by these bias curves although their magnitude may be estimated from the slope of the bias curve. These secondaries have been further estimated by integrating the Møller cross section over the foil thickness and the Rutherford cross section over the solid angle subtended by the mouth of the cup from the bottom of the cup to be of the order of 0.1%.

300-Mev Integrator

Initial testing of the large integrator was done at 35 Mev where the number of backscattered electrons from the lead cup is larger.

No effect was observed due to increasing the pressure in the vacuum chamber surrounding the lead collector until it reached 10⁻³ mm Hg. In all subsequent measurements the vacuum in the shell was maintained at <10⁻⁴ mm Hg.

Figure 11 shows the bias curve obtained at 35 Mev and the effect of placing a carbon plug at the bottom of the cup to decrease the backscattering.

Figure 12 shows the bias curve obtained at 150 Mev with the carbon plug (solid line). At this stage, a secondary-emission monitor was being used and was mounted in the same vacuum as the integrator. Some or all of the bias effect was interpreted as being due to secondary electrons from the monitor entering the integrator and being counted as primary beam. To test this hypothesis a transverse magnetic field sufficient to bend 200-kev electrons out of the integrator aperture was applied between the integrator and the monitor. The result is the dotted curve in Fig. 12. Since the expected effect is to increase R by decreasing the charge collected on the integrator and the opposite effect was observed, the hypothesis was rejected. The bias effect was then interpreted as being due to backscattered electrons from the primary beam and the small reduction in R observed was attributed to penetration of the field into the aperture of the cup. To test this, a series of permanent magnets was clamped between two Lucite disks of diameter equal to the inside diameter of the cup. The resulting field was homogeneous to a few percent and of magnitude 300 gauss. The Lucite was coated with Aquadag and placed at the bottom of the cup. The resulting bias curve (Fig. 13) is consistent with the second hypothesis, and the remaining effect is so small that unless the most extreme accuracy is required the cup may be used at ground potential.

Exposures with x-ray film were made at 250 Mev to determine the amount of ionizing radiation escaping from the cup. A conservative interpretation of the data yields the result that not more than 0.3% of the incident electron beam was lost due to shower penetration. For the parameters of the cup, Eq. (14) predicts 0.1%.

ACKNOWLEDGMENTS

The development and testing of these Faraday cups have been the result of the combined efforts of many people at this Laboratory. Dr. D. D. Reagan, Dr. A. M. Berman, Dr. C. Hsieh, and K. M. Sherwin assisted in various phases of the design and testing; the photographic film exposures were made by Dr. C. M. Newton; J. Helmer designed the automatic slide-back circuit of Fig. 8; Professor W. K. H. Panofsky and Professor W. C. Barber contributed greatly to the over-all program.